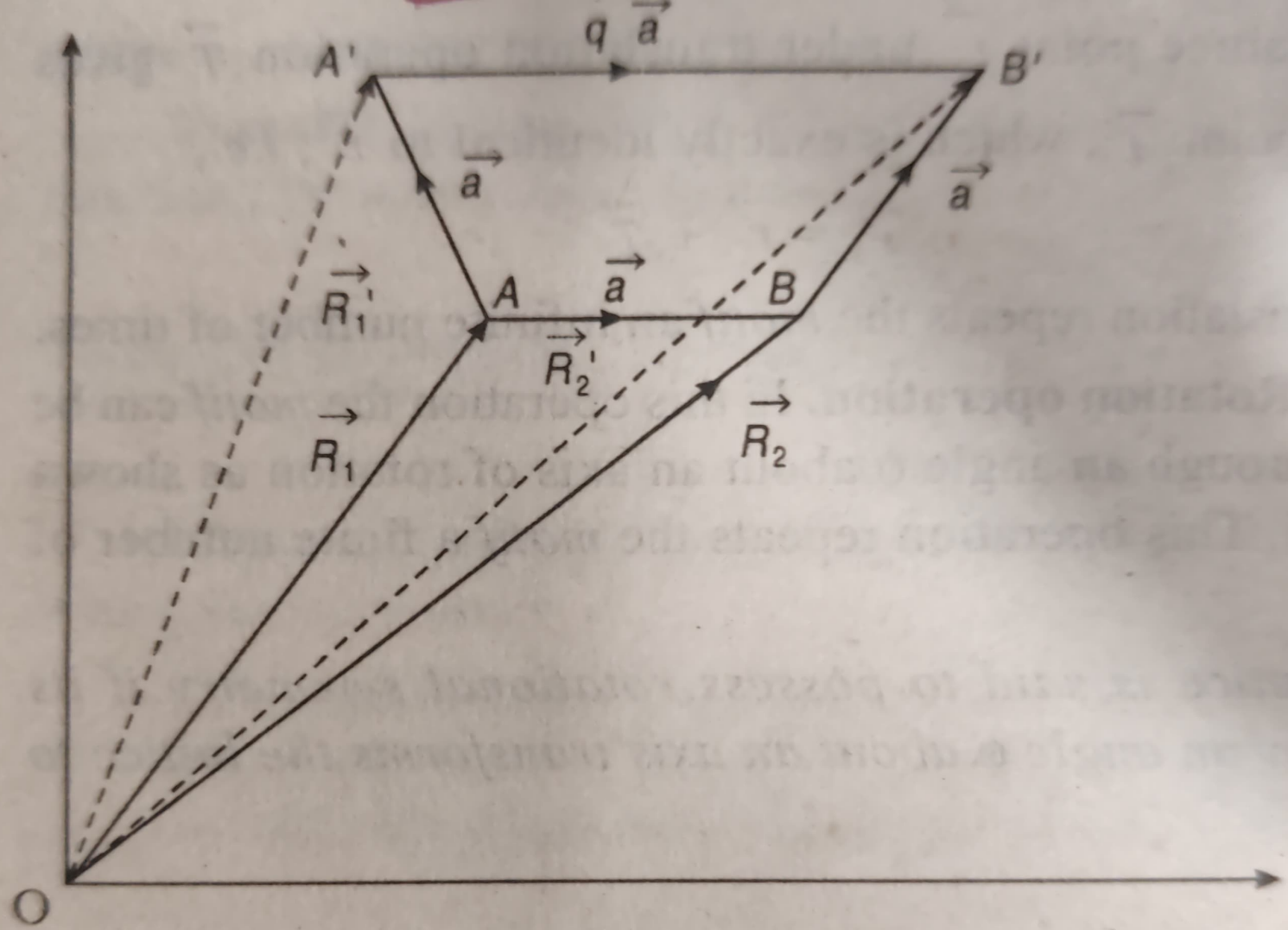

Fig. 1.8

(*b*) Crystal lattice cannot have five fold symmetry. It is not possible to find a lattice that goes into itself under rotations like $2\pi/5$ or $2\pi/7$ or by any other symmetry operation like mirror reflection or inversion. In other words, a *crystal cannot have five fold symmetry*.

To prove it mathematically let $AB = \vec{a}$ be a primitive translation vector and $OA = \vec{R}_1$, $OB = \vec{R}_2$ the two lattice vectors connecting the origin to A and B - the two closest lattice points of the crystal, then

$$|\vec{a}| = |\vec{R}_1 - \vec{R}_2|$$

Now consider a line perpendicular to the plane of the paper and passing through the lattice point A. Let the vector $\vec{a}$ be rotated about this line as axis through an angle $\theta = \frac{2\pi}{n}$ in the anti-clockwise direction, so that it produces a new lattice vector $OA' = \vec{R}_1'$. Similarly, let the vector $\vec{a}$ be rotated about an axis passing through the lattice point B and perpendicular to the plane of the paper through the same angle $\theta = \frac{2\pi}{n}$ in the clockwise direction so that it produces a new lattice vector $OB' = \vec{R}_2'$.

$\therefore$
$$AA' = BB' = |\vec{a}|$$
and
$$\angle BAA' = \angle ABB' = \theta.$$

The quadrilateral is a trapezium with $A'B'$ parallel to AB. Further $A'B'$ must also be an integral multiple of the primitive translation vector $\vec{a}$

or
$$A'B' = q|\vec{a}|$$
where q is an integer.

Now
$$A'B' = q|\vec{a}| = |\vec{R}_1' - \vec{R}_2'|$$
$$= |\vec{a}| + |\vec{a}| \sin\left(\theta - \frac{\pi}{2}\right) + |\vec{a}| \sin\left(\theta - \frac{\pi}{2}\right)$$
$$= |\vec{a}| + 2|\vec{a}| \sin\left(\theta - \frac{\pi}{2}\right)$$
$$= |\vec{a}|[1 - 2\cos\theta]$$

$\therefore$
$$q = 1 - 2\cos\theta \text{ or } |\cos\theta| = \left|\frac{1-q}{2}\right|$$

As $\cos\theta < 1$, the above relation can be satisfied only when $q = 3, 2, 1, 0, -1$.

or
$$\theta = \frac{2\pi}{2}, \frac{2\pi}{3}, \frac{2\pi}{4}, \frac{2\pi}{6}, \frac{2\pi}{1} \quad \text{or } n = 1, 2, 3, 4, 6 \text{ only.}$$

Thus we find that a crystal can have only $\bar{1}, \bar{2}, \bar{3}, \bar{4}$ and $\bar{6}$ fold axis of rotation or *symmetry* and cannot have *five fold symmetry*.

For $n = 2, 3, 4$ and 6 the symmetry is known as *diad, triad, tetrad* and *hexad* respectively.